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LOCALIZATION OF DAMAGE IN BEAM-LIKE STRUCTURES APPLYING TIME-FREQUENCY DISTRIBUTIONS TO MODAL SHAPES OF VIBRATION

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Summary

The paper presents the results of computational studies on damage localization in beam-like structures based on an approach of application of selected time-frequency distributions to modal shapes of vibration. The studies were performed on the results of finite element analysis on determination of modal shapes of vibration of a composite beam. The damage detection and localization procedure was shown on several examples of time-frequency distributions with a discussion of their performance for structural damage detection and localization problems. Additional studies were performed for artificially noised modal shapes in order to investigate an applicability of the proposed approach in noisy conditions. It was shown that application of time-frequency distributions is the effective tool in structural damage detection and localization, and can be a good alternative to the well-known signal processing techniques applied in these problems.

Keywords: structural damage localization, time-frequency distributions, signal processing, modal shapes of vibration

LOKALIZACJA USZKODZEŃ W STRUKTURACH TYPU BELKA Z ZASTOSOWANIEM DYSTRYBUCJI CZASOWO-CZĘSTOTLIWOŚCIOWYCH DO POSTACI WŁASNYCH DRGAŃ

Streszczenie

Artykuł przedstawia wyniki badań obliczeniowych dotyczących lokalizacji uszkodzeń w strukturach typu belka na podstawie podejścia opartym na zastosowaniu wybranych dystrybucji czasowo-częstotliwościowych do postaci własnych drgań. Badania były przeprowadzone na wynikach analizy metodą elementów skończonych dotyczącej wyznaczenia postaci drgań własnych belki kompozytowej. Procedura detekcji i lokalizacji uszkodzeń została pokazana na kilku przykładach dystrybucji czasowo-częstotliwościowych wraz z dyskusją w zakresie ich efektywności w zagadnieniach detekcji i lokalizacji uszkodzeń strukturalnych. Dodatkowe badania zostały przeprowadzone dla sztucznie zaszumionych postaci drgań własnych w celu zbadania stosowalności zaproponowanego podejścia w warunkach zaszumienia. Pokazano, że zastosowanie dystrybucji czasowo-częstotliwościowych jest efektywnym narzędziem przy detekcji i lokalizacji uszkodzeń strukturalnych oraz może stanowić dobrą alternatywę do szeroko znanych technik przetwarzania sygnałów stosowanych w tych zagadnieniach.

Słowa kluczowe: lokalizacja uszkodzeń strukturalnych, dystrybucje czasowo-częstotliwościowe, przetwarzanie sygnałów, postacie własne drgań

1. INTRODUCTION

Structural damage detection and localization based on vibration data has been intensively developed since the eighties of XX century. The first of the developed methods are global and allow for damage detection only. The example of such an approach is the popular measure at this time – Modal Assurance Criterion (MAC), which reflects consistency of modal shapes of damaged and healthy structures, thus is based on the baseline-approach. The detailed description of the MAC as well as its derivatives can be found in [1].

Later, the new approaches were developed which allow localizing damage, thus the new methods become local. In contrast to MAC, where the modal frequencies were taken into consideration, the new methods were based on changes in modal shapes of an inspected structure which reveals much better sensitivity to damage occurred in a structure. The first methods were based on Modal Strain Energy (MSE) techniques developed by Stubbs et al. [2]. Later, the method was enhanced by several research teams by introducing MSE-based damage indices and indicators [3-7]. The MSE-based approach uses second-order derivative of modal shapes to localize damage which allows increasing its sensitivity with respect to MAC-based methods, however MSE-based methods are still based on baseline data, thus need a healthy structure or a model of such a structure to compare with the damaged one for the purpose of damage detection.

In order to overcome the problem with a baseline several new methods were developed. One of such methods is the Gapped Smoothing Method (GSM) developed by Ratcliffe [8,9]. This method is based on estimation of local changes of the stiffness of a tested structure due to an occurred damage, which results in tiny changes in magnitudes of modal shapes, and does not need a baseline data. Another baseline methods used in structural damage detection and localization consist of the methods based on derivation of modal shapes [10-12], fractal dimension [13-15], and others. Although the above-described methods do not need a baseline and reveal quite high sensitivity to disturbances in modal shapes resulted by damage in some cases for higher-order modal shapes they give false peaks in the locations of extrema.

A special attention should be paid to the wavelet-based methods. The wavelet transform (WT) allows for achievement of even better sensitivity than the previously described methods, it does not need baseline data, and, depending on an applied wavelet function, can give a very clear result of damage localization and identification. The first applications of WT for structural damage detection and localization problems were based on the continuous wavelet transform (CWT) which makes application of almost arbitrary basis functions possible (see e.g. [16-19]). Another approaches use the discrete wavelet transform (DWT) [20,21] and the stationary wavelet transform (SWT) [22,23] in order to achieve better localization properties, remove undesirable effects and reduce computational time. An overview on wavelet-based methods applied for structural damage localization and identification with numerous examples can be found in [24].

The problem of application of WT-based methods in structural damage localization and identification problems is that the result of such an analysis significantly depends on a type and parameters of a selected basis function. Additionally, there are no strict rules of selection of such a function, i.e. the selected wavelet always depends on a character of a transformed signal and a type of information desired after performing an analysis. In order to overcome the problem of selection of a wavelet type and its parameters for proper localization and identification of damage the time-frequency distributions (TFDs) can be applied. Similarly as WTs, TFDs return a result in two domains simultaneously. However, in contrast to WTs, TFDs do not use a basis function, but is based on autocorrelation performed on an analyzed signal. Depending on a kernel of a TFD in the ambiguity function, various TFDs are sensitive to various components in an analyzed signal. Appropriate selection of a type of TFD may give relevant results in structural damage localization and identification problems, comparable to those obtained using WT-based methods.

In this paper, the structural damage localization problem was considered based on selected modal shapes of vibration of a cantilever beam obtained from finite element (FE) analysis. The analysis was performed using selected TFDs, and the performance of these TFDs was shown and discussed.

2. PROBLEM DESCRIPTION AND DATA PROCESSING

2.1. Properties of time-frequency distribution

TFDs were developed with a dedication for an analysis of signals defined in time domain. The resulting spectrogram is presented in the form of a time-frequency scatter plot which allows for observation of an evolution of signal components in both domains simultaneously. This approach can be adapted to the analysis of modal shapes of vibration by replacing a signal defined in time with a signal (modal shape) defined in space (length of a beam in the considered case). Therefore, the obtained spectrogram represents such a signal in space-pseudo-frequency domains. The pseudo-frequency denotes here the "waviness" of a modal shape and increases with an increase of a mode number, and thus, a frequency of vibration.

Various TFDs reveal better sensitivity to particular types of components of signals which was investigated and discussed in [25]. Following this, it is suitable to select TFDs with a sensitivity focused on pulse components, since the response of a simulated crack consists of a sudden change of a signal value at the location of this crack.

The selection of TFDs can be performed by the analysis of properties of TFDs. One can neglect the TFDs in this study which have interferences (e.g. high-magnitude cross-terms in the space-pseudo-frequency representation). Such properties has, for example, the fundamental and the oldest TFD – Wigner-Ville distribution (WVD) given by:

$$\mathrm{WVD}_x(t,f)=\int_{-\infty}^{+\infty} x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)\cdot\exp(-j2\pi f\tau)\,\mathrm{d}\tau\,, \qquad (1)$$

where t is a time, f is a frequency, x^* is a complex conjugate of a signal x, τ is a time lag. This TFD belongs to the Cohen's class of TFDs which is given in general form by:

$$\mathrm{CCD}_x(t,f)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\phi(t-t',f-f')\,\mathrm{WVD}_x(t',f')\,\mathrm{d}t'\,\mathrm{d}f'=$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Phi(\tau,\upsilon)A_x(\tau,\upsilon)\exp[j2\pi(t\upsilon-f\tau)]\,\mathrm{d}\tau\,\mathrm{d}\upsilon\,, \qquad (2)$$

where

$$\phi(t,f)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Phi(\tau,\upsilon)\exp[j2\pi(t\upsilon-f\tau)]\,\mathrm{d}\tau\,\mathrm{d}\upsilon\,, \qquad (3)$$

$$A_x(\tau,\upsilon)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)\cdot\exp(-j2\pi\upsilon t)\,\mathrm{d}t\,, \qquad (4)$$

is the ambiguity function and $\Phi(\tau,\upsilon)$ is a kernel function in an ambiguity domain specific for particular types of distributions.

The first TFD with desirable properties for the considered structural damage localization problem is the Rihaczek distribution (RD) with a kernel function given by:

$$\Phi_{\mathrm{RD}}(\tau,\upsilon)=\exp(j\pi\upsilon\tau). \tag{5}$$

This distribution is suitable for damage localization problems due to its ability of perfect localization of pulse components, i.e. if $x(t) = \delta(t-\tau)$ then $R_x(t,f) = \delta(t-\tau)$ [26]. The known drawbacks of RD in signal processing, namely appearance of negative or even complex values [26], do not disqualify this distribution from the consideration of structural diagnostics problems. Taking into consideration its absolute values is enough to detect and localize damage. The RD has several derivative distributions with similar properties: the Margenau-Hill distribution (MHD) and Page distribution (PD) with kernels defined as:

$$\Phi_{\mathrm{MHD}}(\tau,\upsilon)=\cos(\pi\upsilon\tau), \tag{6}$$

$$\Phi_{\mathrm{PD}}(\tau,\upsilon)=\exp(j\pi\upsilon|\tau|), \tag{7}$$

which are the real part and causal part of RD, respectively [26]. Their kernels defined in the ambiguity domain are similar with respect to RD. Such properties also have the smoothed and reassigned versions of the mentioned TFDs.

Analyzing the other TFDs of Cohen's class the pulse components are not perfectly localized in space and pseudo-frequency domains which may affect negatively on ability of detection and localization of damage, especially in the conditions of noise presence. These effects are presented based on the considered problem in the next section.

2.2. Numerical model

The developed TFD-based structural damage localization approach was tested on results of numerical modal analysis performed on a 3D model of a 12-layered composite cantilever beam with the lay-up given by the following structural formula: $[0/60/-60]_{2S}$. The model of a beam was prepared in the Marc/Mentat® commercial software. The dimensions of the modelled beam were as follows: the length of 0.2 m, the width of 0.01 m and the thickness of 0.0024 m. The model was meshed using hexagonal 8-node elements. The damage was simulated in the form of the through-the-width crack at the distance of 0.15 m from the clamped side with a gap size of 0.0005 m and the depth of 0.0004 m (the thickness of two layers). Further details on model preparation and material properties can be found in [27]. For increasing the accuracy of numerical solution the region of simulated crack was meshed with smaller elements, however, in further studies the response from equidistant nodes only was considered. Moreover, the location of a crack was simulated in such a way that its position does not interrupts with the considered node in order to avoid changes in the node resulting from physical absence of material in the thickness. For further studies only the nodes located in the direction of the length on its top and from the middle along the width direction of a beam were taken into consideration. Therefore, the considered data are one-dimensional.

The numerical simulation was performed in order to acquire modal shapes of vibration. The acquired data consisted of nodal displacements for selected modal shapes. 256 equidistant nodes were collected for each modal shape. The second bending modal shape was considered during structural damage localization study due to the difficulties of obtaining high-magnitude differences for the first modal shape.

3. RESULTS OF CRACK LOCALIZATION

3.1. Crack localization using proposed approach

The crack localization study was performed in Matlab® using the Time-Frequency Toolbox developed by Auger et al. [28]. The acquired modal data was imported into Matlab® environment and transformed using selected TFDs. In order to validate the results of the discussion on properties of various TFDs presented in section 2.1, the second modal shape was visualized in the form of space-pseudo-frequency representations using WVD, RD, MHD, PD discussed earlier as well as using Choi-Williams distribution (CWD):

$$\Phi_{\mathrm{CWD}}(\tau,\upsilon)=\exp\left(-(2\pi\upsilon\tau/\sigma)^2\right), \tag{8}$$

where σ is a temporal scaling parameter, and Zhao-Atlas-Marks distribution (ZAMD):

$$\Phi_{\mathrm{ZAMD}}(\tau,\upsilon)=\exp\left(-(2\pi\upsilon\tau/\sigma)^2\right)\cos(2\pi\beta\tau), \tag{9}$$

where β is a spectral scaling parameter. Due to appearance of complex numbers in the obtained representations only their real parts were considered. The pseudo-frequency parameter was set to 500. The resulting representations are presented in Fig.1.

The obtained results confirm the properties of the applied TFDs. The results obtained using WVD show that the pulse component was not detected clearly. The sudden change of values magnitude at 0.15 m (Fig.1a) cannot be considered as a result of detection unambiguously. The results obtained for RD, MHD and PD show the location of the crack very clear (see Figs.1b-d), which justifies the perfect localization of pulse components for these TFDs. Due to similar mathematical formulation of kernels of these TFDs the resulting space-pseudo-frequency representations look very similar. In the case of CWD the ability of localization of a pulse component in the analyzed signal is poor, therefore the component at the crack location is blurred (see Fig.1e).

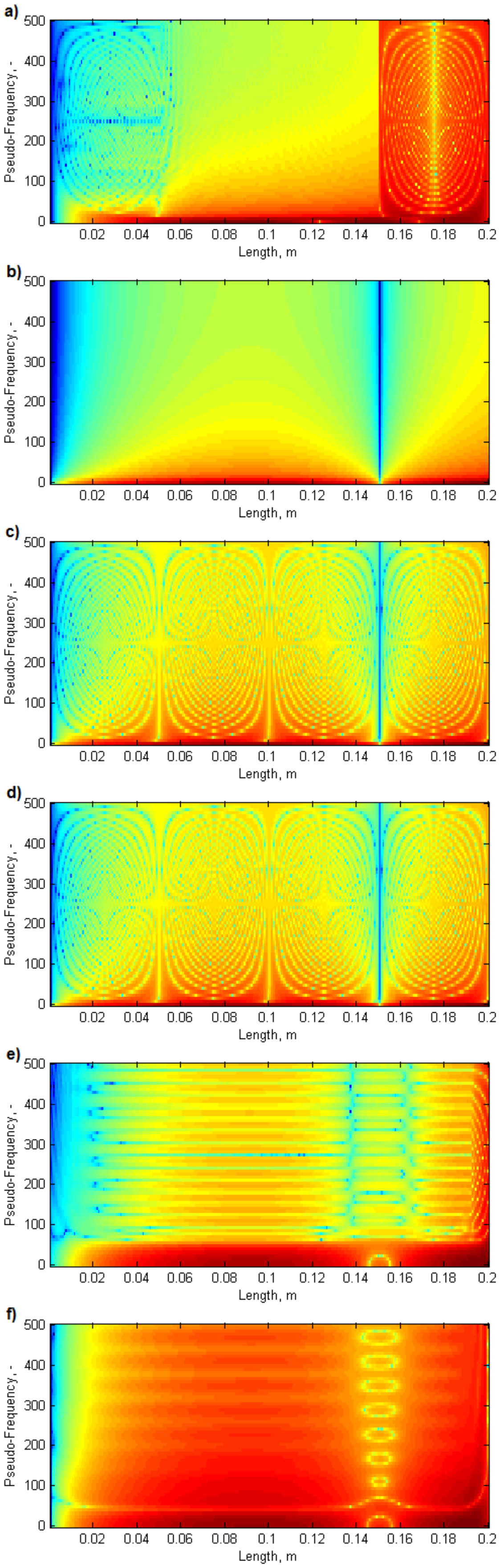

Fig.1. Space-pseudo-frequency representations of the considered modal shape using a) WVD, b) RD, c) MHD, d) PD, e) CWD, f) ZAMD

Better result can be obtained when applying ZAMD (see Fig.1f), however the localization of a pulse component is still unsatisfactory with respect to RD and similar formulations of TFDs.

3.2. Influence of noise

In order to simulate real measurement conditions and evaluate robustness to noise of the proposed damage localization procedure the modal shape obtained from FE analysis was noised with various signal-to-noise ratios (SNR). The modal shape values were noised using the amplitude-based SNR relation given by:

$$\mathrm{SNR}_{\mathrm{dB}} = 20\lg\left(\frac{A_f}{A_n}\right), \tag{10}$$

where A_f and A_n denote the root mean square (RMS) of a signal and a noise, respectively.

Due to the perfect localization property of the RD and its derivatives, confirmed in the case study in the previous subsection, the RD and MHD were considered in the following study (only MHD was chosen due to the similarity between the representations obtained using MHD and PD – see Fig.1c and Fig.1d). The analysis of influence of a noise on the ability of localization the damage position shows a good robustness to the noise. The damage is still detectable at 30-40 dB of SNR. The results for SNR of 30 dB and 40 dB for both distributions are presented in Fig.2.

4. CONCLUSIONS

The following study presents an adaptation of time-frequency distributions of the Cohen's class to vibration-based structural damage localization problems. The adaptation was performed based on replacement of the time domain by the space domain, while the frequency domain was substituted by the pseudo-frequency domain which describes the waveform of the modal shape. The properties of the selected time-frequency distributions were investigated, and based on this analysis the most sensitive time-frequency distributions to structural damage in beams were chosen. Further, the theoretical analysis was verified by the performed experimental studies on numerically acquired modal shape of vibrations of a cantilever composite beam. The obtained results showed high precision in detection and localization of the crack in the modeled beam when Rihaczek distribution and its derivatives were applied. This is due to the property of perfect localization of pulse components of these distributions. As a counterexample, other distributions from the Cohen's class were used for damage detection and localization. It was shown that the applied distributions reveal poor detectability and localization ability of damage. Finally, the noise robustness was tested by artificially noising of the considered modal shape. The obtained results show that the damage is still detectable and localizable even at 30 dB of noise which means that the vibration signal acquired in traditional way using accelerometers is still relevant for performing damage detection and localization procedures on it.

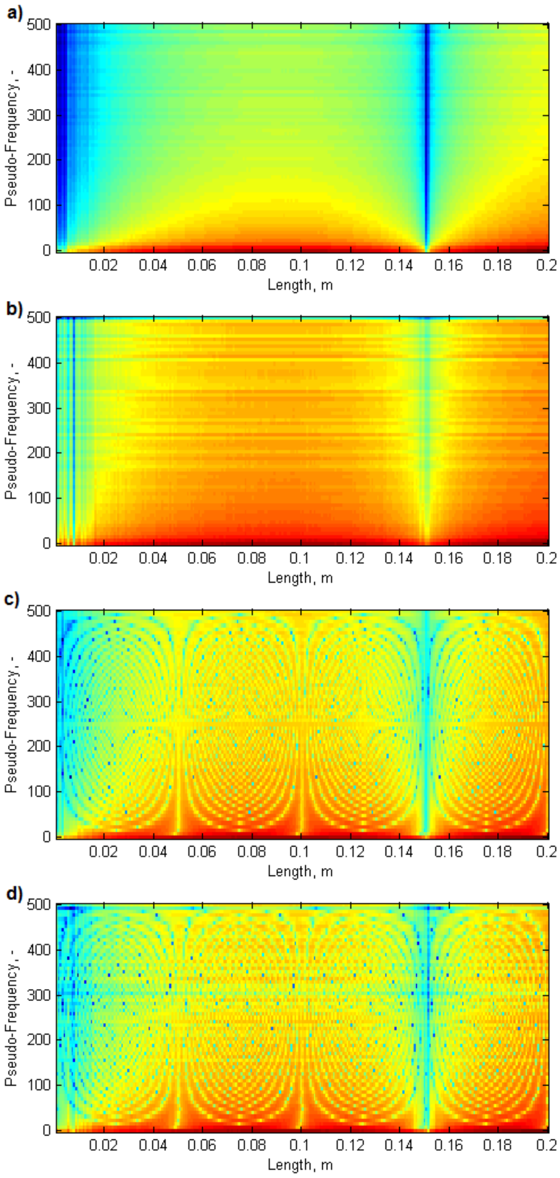

Fig. 2. Space-pseudo-frequency representations of the considered modal shape using RD with a) 40 dB, b) 30 dB of SNR, and MHD with c) 40 dB, d) 30 dB of SNR

The approach of the structural damage detection and localization based on modified Rihaczek-like representations of time-frequency distribution reveals high-accuracy results comparable to those obtained using the wavelet-based approach (cf. [27]). In contrast to the wavelet-based approaches, the proposed one does not require selection of a type and parameters of an applied basis function, since this function does not exist in mathematical formulations of time-frequency distributions. As mentioned before, there are neither rules nor mathematical fundamentals of selection of a basis in wavelet-based approaches, however the proper selection of a basis is crucial for accuracy of detection and localization of damage (see comparative studies in [25]). Application of the approach based on time-frequency distributions allows avoiding this problem without a loss of accuracy of damage detection and localization.

Further studies in the area of structural damage detection and localization will be focused on generalization of the presented approach to the two-dimensional problems, and evaluation of its sensitivity to various types of structural damage.

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