STABLE DISTRIBUTIONS, GENERALISED ENTROPY, AND FRACTAL DIAGNOSTIC MODELS OF MECHANICAL VIBRATION SIGNALS

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Abstract

Vibrodiagnostic analysis of wearing and/or defects of complex rotating systems confirms the presence of non-linear, nonstationary and multiscale properties as well as long-term correlations of real signals. The recorded time series of vibrations are often of an impulse character. Probability distributions are different than Gaussian distributions and exhibit heavy-tails. These are important sources of multifractal dynamics, requiring advanced, data-based modelling methods. The reliable numerical algorithms, used for calculations of functions of stable distributions and multifractal properties, were applied in the approach presented in the hereby paper. Relations between parameters of stable distributions and singularity spectra indicate the possibility of applying both methods for modelling mechanical vibrations signals in diagnostics of complex systems. The performed investigations confirmed the possibility of modelling and assessing the observed states of the powertrain of vehicles with SI engines, on the bases of parameters of alpha-stable distribution (ASD) and parameterised entropy of mechanical vibrations signals.

Keywords: vibrodiagnostics, multifractality, stable distributions, parameterised (generalised) entropy

ROZKŁADY STABILNE, ENTROPIA UOGÓLNIONA I FRAKTALNE MODELE DIAGNOSTYCZNE SYGNAŁÓW DRGAŃ MECHANICZNYCH

Streszczenie

Wibrodiagnostyczna analiza zużycia i / lub wad złożonych układów wirujących potwierdza obecność nieliniowych, niestacjonarnych i wieloskalowych właściwości oraz długookresowe korelacje sygnalozywisych. Rejestrowane szeregi czasowe drgań mają często charakter impulsowy. Rozkłady prawdopodobieństwa odbiegają od rozkładów Gaussowskich i wykazują gruboogonowość. Są to ważne źródła dynamiki multifraktalnej, wymagające zaawansowanych metod modelowania bazującego na danych. W podejściu przedstawionym w pracy wykorzystano niezawodne algorytmy numeryczne służące do obliczania funkcji stabilnych dystrybucji i cech multifraktalnych. Relacje między parametrami stabilnych rozkładów i widmami osobliwości wskazują na możliwość zastosowania obu metod do modelowania sygnałów drgań mechanicznych w diagnostyce układów złożonych. Wykonane badania potwierdziły możliwość modelowania i oceny obserwowanych stanów układu napędowego pojazdu z silnikiem o zapłonie iskrowym, na podstawie parametrów rozkładów alfastabilnych (ASD) gęstości prawdopodobieństwa i entropii parametryzowanej sygnałów drgań mechanicznych.

Słowa kluczowe: wibrodiagnostyka, multifraktalność, rozkłady stabilne, parametryzowana (uogólniona) entropia

1. INTRODUCTION

The analysis of time series of mechanical vibrations, performed by means of computers of the higher and higher computational power and advanced numerical algorithms, constitutes the new quality in modelling complex systems. Multilevel dynamics, determinism and randomness are the phenomena observed at the analysis of non-linear mechanical vibration signals, which indicate the need of a simultaneous application of mechanics and statistics rules. The control and diagnostics of more and more complex systems, containing several non-linear subsystems - which cannot be separated without changing their dynamic properties - require more accurate non-linear models [3,4,14,17]. The data-driven method allows obtaining the time series statistic model and identification of deviations of the changes in monitored system dynamics.

Several methods are based on the classic theory of random signals analysis, however a lot of reasons indicate the necessity of using alpha-stable distributions instead of normal distributions at describing complex systems. The stable distributions are asymptotically - according to the generalised central limit theorem (GCLT) - the only

distribution of the sum of independent and identical distributions of random variables.

The analysis of mechanical vibration signals of complex systems confirms the presence of long-term correlations and scaling properties, not only the non-linear and nonstationary properties. When describing the behaviour of real dynamic systems by means of the power law with a fractional exponent, it means with the fractal measure, the possibility of their modelling in the multifractal domain is obtained [13]. The fractal, box dimension *D* measure is defined as the measure of the curve being the diagram of the signal under consideration [8], based on the power dependence: $L \sim \varepsilon^{-D}$, where *L* is the minimal number of boxes in ε scale, covering the given time series. The single fractal measure is the averaged information, related to the analysed range of scales. The description of the multifractal dynamics, it means of several interlaced fractals, requires the determination of the singularity spectrum. It is equivalent to the description of local properties - of representing it time series - by means of the multifractality spectrum. The spectrum obtained as the segmentation result, is the histogram of local regularity degrees of the investigated signal. Scaling of the signal fluctuation function, of the partition function its probabilistic measure as well as scaling the parameterised Renyi entropy [22, 29], constitutes the base of the multifractal formalism. The additive Renyi entropy constitutes the generalisation of the informative Shannon entropy, used in the information theory for assessing uncertainties of dynamic systems [6, 10, 30]. The generalised fluctuation and partition function exponents as well as the Renyi entropy fractal measure are related to the multifractal spectrum by means of the Legendre transformation [11].

The multifractal scaling is observed in several time series of mechanical vibrations generated in dynamic systems. The heavy-tail probability distribution, typical for stable distributions, as well as the long-term correlations are two basic multifractality sources [12]. Examples of applications of alpha-stable models in health monitoring are given in papers [7, 31, 34, 35-38]. Various multifractal methods were used in modelling dynamic systems by the authors of papers [18-21, 23, 24, 27, 39]. In the method for rolling bearings diagnosis based on feature fusion the advantages of multifractal detrended fluctuation analysis (MF-DFA) and ASD were applied, to achieve an intelligent monitoring [33].

The results concerning the analysis of mechanical vibration signals in the vehicle powertrain are presented in this paper. The diagnostic data-driven models (that combine multifractal features with stable distribution features) using the parameters of stable distributions and multifractal measures of singularity spectra, based on the parameterised entropy, were proposed and verified. The theories

of alpha-stable distributions and generalised entropy in modelling of signals having the multifractal character are described and analysed in chapters 2 and 3, respectively. Chapter 4 contains the results of investigations of mechanical vibration signals in the vehicle powertrain. Conclusions are presented in chapter 5.

2. ALPHA-STABLE DISTRIBUTION. SIMULATION STUDY

The recorded time series of complex rotating systems exhibit the impulse-like nature and fluctuations [28]. Probability distributions often deviate from the Gaussian distribution and exhibit heavy tails. ASD family received interest due to its success in modelling data, which are too impulsive to accept the normal distribution. The lack of closed formulas for densities and distribution functions for all but a few stable distributions Gaussian, Cauchy and Levy were a major problem in using stable distributions in technical diagnostics. There are now reliable computer programs to compute stable densities, distribution functions and parameters [25]. With these programs, it is possible to apply stable models in a variety of practical problems. The alpha-stable distribution is described by its characteristic function:

$$\varphi(t) = \exp\{j\delta t - \gamma |t|^{\alpha} [1 + j\beta \operatorname{sign}(t)\omega(t,\alpha)]\} \quad (1)$$

where

$$\omega(t,\alpha) = \begin{cases} -\tan\dfrac{\alpha\pi}{2} & if \quad \alpha \neq 1 \\ \dfrac{2}{\pi}\log|t| & if \quad \alpha = 1 \end{cases} \quad (2)$$

Data modeling using stable distributions require four parameters to their full description. These parameters are as follows:
• stability index $\alpha \in (0,2]$;
• a skewness parameter $\beta \in [-1,1]$;
• a scale parameter $\gamma > 0$;
• a location parameter $\delta \in$ R.

Index α describes impulsive character of distribution and thickness of distribution tail. For $\alpha = 2$; 1 and 0,5 the Gaussian; Cauchy and Levy distributions can be modeled, respectively. For $\alpha < 2$ the decay distribution follows a power-laws. Skewness parameter $\beta = 0$ implies that the distribution is symmetric. Negative or positive β implies that the distribution is skewed to the left or to the right respectively. The parameters γ and δ are similar to the variance and the mean of a normal distribution. Time series of simulated alpha-stable signals are shown in Fig. 1. Fig. 2 shows effect of a stability index on stable distribution.
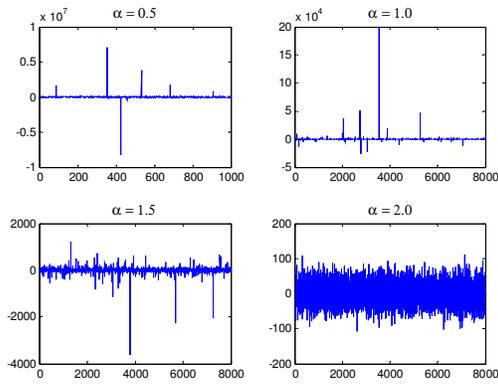
Fig. 1. Time series of simulated alpha-stable
signals: α, $\beta= 0$, $\gamma=20$, $\delta=0$
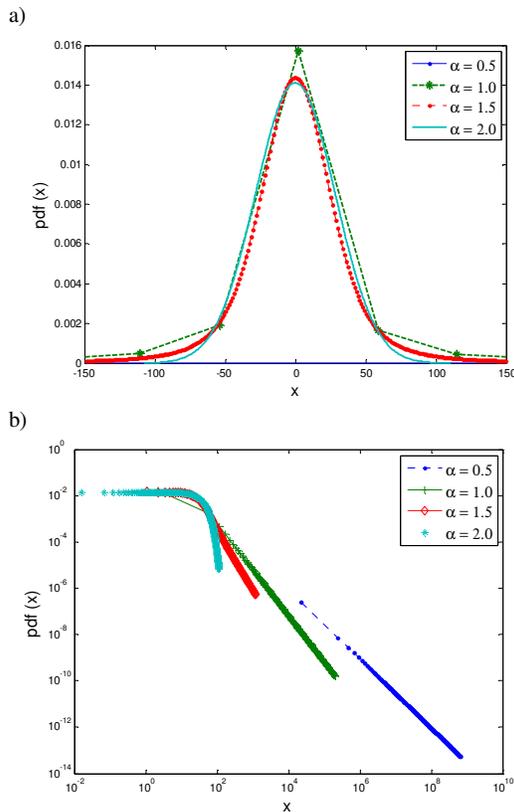
a)

b)

Fig. 2. PDF (a) and right tail PDF (b) of
simulated alpha-stable signals

3. MULTIFRACTAL METHODS IN DATA MODELLING

The relationship between the parameters of stable distributions and the multifractal spectra indicates the possibility of using both methods in modelling the mechanical vibration diagnostic signals of complex systems [1]. The base of various methods of the multifractal formalism is scaling the measures assumed in relation to the analysed measures signal, by means of which the segmentation is performed.

3.1. Algorithm of detrended fluctuations analysis MF-DFA

Selfafinic time series (or exhibiting such property after summing), are described by the Hurst exponents [9] related to the fractal measure D ($D=2$-H). The numerically simple estimation of the Hurst exponent allows the analysis of MF-DFA method [2].

Step I. Defining of the cumulated, centred, random variable $X(i)$ for the time series x_i with the estimator of the expected value.

Step II. Dividing the centred cumulation sum into separate segments of a length ε, starting this dividing once from the beginning and for the second time from the end. In such way $N_\varepsilon = 2\,\mathrm{int}\,\dfrac{N}{\varepsilon}$ segments ε are obtained. For each segment v the profile represented by the polynomial $x_v(i)$ is obtained by the least square method, and detrending is performed.

Step III. Averaging of calculated variances $F^2(\varepsilon,v)$ for all segments v and the determination of the fluctuation function $F_q(\varepsilon)$ of the order equal q.

Step IV. Repeating steps 2 - 3 for various time scales ε and the analysis of fluctuation power dependencies in two-logarithmic scale, to determine the generalised Hurst exponent $H(q)$.

The procedure realised in the algorithm leads to determining the fluctuation power dependency of the q order:

$$F_q(\varepsilon) = \left\{ \frac{1}{2N_\varepsilon} \sum_{i=1}^{2N_\varepsilon} \left[F^2(\varepsilon,v) \right]^{\frac{q}{2}} \right\}^{\frac{1}{q}} \sim \varepsilon^{H(q)} \qquad (3)$$

The exponent $H(q)$ is a decreasing function. For negative values of the q order, the generalised Hurst exponent describes scaling properties in segments of a low fluctuation level, while when the positive values of the q order are considered, segments of high variances are shaping the fluctuation function.

The generalised scaling exponent $\tau(q)$ and the multifractal spectrum $f(h)$ are determined by equations:

$$\tau(q) = qH(q) - 1 \qquad (4)$$

$$f(h) = qh - \tau(q) \qquad (5)$$

where the singularity exponent h:

$$h = \frac{d}{dq}\tau(q) \qquad (6)$$

Multifractal spectrum $f(h)$ is a convex function. Pairs: $[q,\ \tau(q)]$ and $[h,\ f(h)]$ are joined by the Legendre transformation.

3.2. Segmentation algorithm according to the multiscale energy distribution

The alternative way of signals segmentation constitutes the description of local properties on the basis of the probabilistic measure [40].

Step I. Dividing the time series into segments in scale ε and the discrete probability distribution

$$P_i(\varepsilon) = \frac{S_i(\varepsilon)}{\sum S_i(\varepsilon)} \qquad (7)$$

where $S_i(\varepsilon)$ is the sum of vibrations amplitudes in the i-th segment, and $\sum S_i(\varepsilon)$ is the sum of all amplitudes of the recorded signal.

Step II. Determination of the Shannon entropy:

$$I(\varepsilon) = -\sum_{i=1}^{N} P_i(\varepsilon) \ln P_i(\varepsilon) \qquad (8)$$

Step III. Parameterisation of the entropy by means of parameter q dividing various periods of the signal, according to the probability value of their occurrence.

$$I_q(\varepsilon) = \frac{1}{1-q} \ln \sum_{i=1}^{N} P_i(\varepsilon)^q \qquad (9)$$

Step IV. Determination of the Renyi entropy exponent

$$D(q) = -\lim_{\varepsilon \to 0} \frac{I_q(\varepsilon)}{\ln \varepsilon} \qquad (10)$$

Exponent $D(q)$, which is not growing function of variable $q \neq 1$, describes the way of scaling of the dynamic state measure, it means the Renyi entropy:

$$I_q(\varepsilon) \sim \ln \varepsilon^{-D(q)} \qquad (11)$$

The Shannon entropy assumes equivalence of data, independently of their abundance determined by the probability density distribution function. The Renyi entropy for $q \gg 0$ exhibits the most probable occurrences, while $q \ll 0$ it relates to occurrences forming tails of the probability density distributions [32].

3.3. Relations of the generalised sum of the probabilistic multifractal measure

In accordance with the multifractal formalism, the generalised sum of the probabilistic multifractal measure, determined by equation (12) is of a power law character within a limit: $\varepsilon \to 0$

$$Z(q,\varepsilon) = \sum_{i=1}^{N} P_i(\varepsilon)^q \sim \varepsilon^{\tau(q)} \qquad (12)$$

The described scaling relations are listed in Table 1.

Table 1. Scaling relations in the multifractal formalism

	Fluctuation	Parameterised entropy	Generalised sum
Function	$F_q(\varepsilon) \sim \varepsilon^{H(q)}$	$I_q(\varepsilon) \sim \ln \varepsilon^{-D(q)}$	$Z(q,\varepsilon) \sim \varepsilon^{\tau(q)}$
Exponent	$H(q)$	$D(q)$	$\tau(q)$

The relation between the Renyi multifractal measure $D(q)$ and the generalised multiscaled exponent $\tau(q)$:

$$\tau(q) = D(q)(q-1), \qquad (13)$$

results from the scaling relations in the multifractal formalism. Thus the Renyi entropy of the order $q \neq 1$, can be estimated on the basis of the equation:

$$I_q(\varepsilon) = -D(q) \ln \varepsilon = \frac{\tau(q)}{1-q} \ln \varepsilon \qquad (14)$$

assuming the smallest applied observation scale of signal ε.

In specific cases the Renyi measure is reduced to other fractal measures. The box dimension corresponds to $q=0$, the information measure - corresponding to the Shannon entropy definition - is obtained for $q \to 1$, while the correlation measure being the probability of finding the pair of points of the phase attractor (reconstructed on the time series base) in the distance smaller than the determined distance, is the consequence of assuming $q=2$.

The entropy span determined for segments of the highest and smallest energy of vibration signal, representing the most probable occurrences as well as the occurrences forming tails of the density probability distributions determined by the parameter $\Delta I_q(\varepsilon)$:

$$\Delta I_q(\varepsilon) = I_{-q}(\varepsilon) - I_q(\varepsilon), \qquad (15)$$

is the measure of the multifractal level of the analysed time series. Multifractality level:

$$\Delta = h_{max} - h_{min}, \qquad (16)$$

where h_{max} and h_{min} are singularities corresponding to the maximum and the minimum fluctuation of the observed signal, respectively, represents heterogeneity of the observed signal. Spectra and parameterised entropies of the simulated alpha-stable, symmetric time series are shown in Fig. 3 and 4.
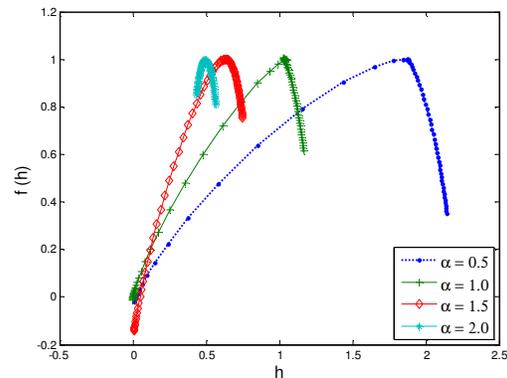

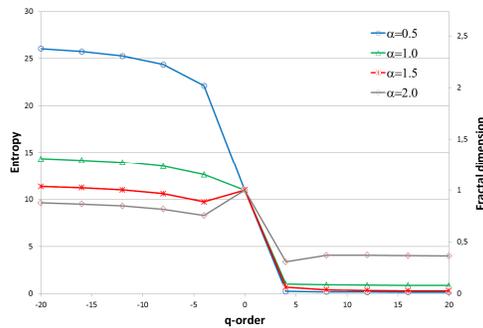Fig. 3. Multifractal spectra of simulated alpha-stable signals

Fig. 4. Parameterised entropy and the fractal dimension of the entropy of simulated alpha-stable signals

4. CASE STUDY OF TIME SIGNALS OF MECHANICAL VIBRATIONS IN THE VEHICLE POWERTRAIN

Signals of accelerations of mechanical vibrations originated from the monitoring of the vehicle powertrain were recorded during investigations [15]. Successive measurements performed in equal time intervals, during road tests of the vehicle with S.I. engine 1.4 l represented values in the experimental data set. Experiments generated, after the angular resampling, time series consisting of 20 rotations of the crank shaft. Each time series of vibrations accelerations during 1 work cycle of the engine contained 3600 signal samples in the determined for the test work conditions. Apart from the signal of the acceleration of vertical vibrations of the main gearbox housing, voltage from the sensor of the crankshaft position and voltage from the sensor of the throttle position were also recorded. Accelerations of vibration signals of the powertrain were processed by means of the Bruel & Kjaer sensors type IEPE No. 4514. Signals were recorded by means of the portable data recording device, Bruel & Kjaer PULSE type 3560E with the sampling frequency of 65536 Hz.

The resampled signal of a length of 72000 samples were divided into segments of the same length and the scale range selection within limits: $\varepsilon \in (1/3600, 1/4)$. The program of investigations included various maintenance states, being the effect of mechanical defects, grouped into classes called symbolically: no-fault C1, initial wearing C2, and serious defect C3. Gearboxes just before a failure or qualified for repairs or for exchanging elements were rated into the serious defect group. Typical averaged empirical and theoretical probability density functions of gearbox vibration signals and their right tails in the tested classes, are shown in Fig. 5 and Fig. 6, respectively.
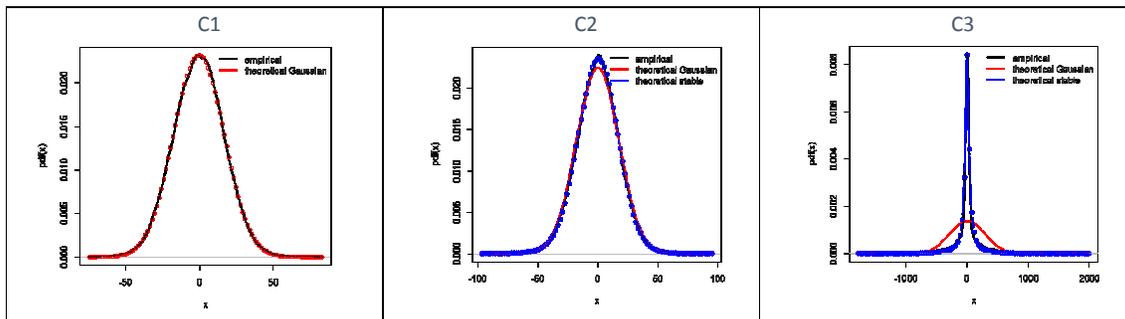

Fig. 5. PDF of gearbox vibration signals in tested maintenance states
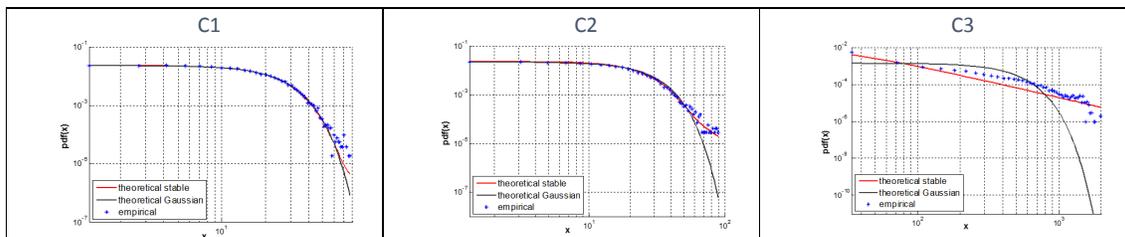

Fig. 6. Right tail PDF of gearbox vibration signal in tested maintenance states

A divergence of empirical distributions with the normal distribution was increasing along with the fault degree of the gearbox. During the goodness-of fit tests the conformity with the null hypothesis - assuming the normal distribution - was rejected for initial wear and serious defect. The verification of matching the alpha-stable model of empirical distributions was performed, after the preliminary graphical assessment, by means of the Anderson-Darling test at the significance level 0.05. In no-fault condition the p-value would be 0.11, which is not significant at the 0.05 level. In other tested maintenance states p-values would be 0.0005. There are several methods and algorithms of estimating the index and parameters of alpha-stable distributions on the basis of experimental data [5]. The results of quantile method, applied in investigation, are presented in Table 2.

Table 2. Averaged parameters of alpha-stable distributions of gearbox vibration signals in tested maintenance states

	good	initial wear	serious defect
α	1,99	1,90	0,77
β	-0,03	-0,23	-0,01
γ	11,92	11,83	34,76
δ	-0,46	-0,05	1,94

Successive diagnostic features were obtained due to the transformation of signals from the time domain to the singularity domain. Multifractal spectra and parameterised entropies in a similar fashion as probability distributions of the tested gearbox signals differ in placements and in shapes. The average results of experimental tests and multifractal analysis are shown in Fig. 7 and 8.
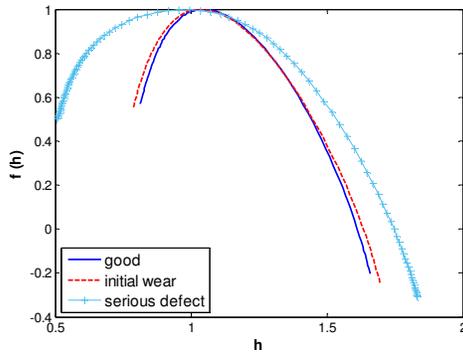

Fig. 7. Singularity spectra of gearbox vibration signals in tested maintenance states
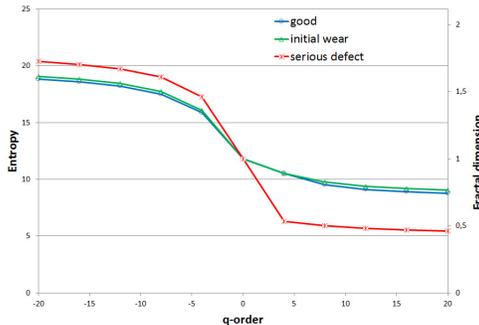

Fig. 8. Parameterised entropy and the fractal dimension of the entropy of gearbox vibration signals in tested maintenance states

Defined spectrum and entropy parameters are presented in Table 3. Parameters of stable distributions, measures of singularity spectra and generalised entropy were tested as diagnostic features allowing to classify the simulated maintenance states of the tested powertrain.

Table 3. Averaged parameters of multifractal spectra of gearbox vibration signals in tested maintenance states

	good	initial wear	serious defect
Δ	0,85	0,90	1,32
$\Delta I_{10}\,(1/3600)$	8,54	8,53	13,58
$\Delta I_{15}\,(1/3600)$	9,70	9,65	14,58
$\Delta I_{20}\,(1/3600)$	10,07	10,01	14,92

The final classification was carried out by using 3-dimensional vector of defects detection of coordinates corresponding to features: stability index - α, scale parameter - γ and parameterised Renyi entropy - ΔI_q for order $q \geq 10$. Index α describes impulsive character of signal and thickness of distribution tail. Parameter γ is a scale parameter and coordinate ΔI_q represents span of proportional abundance determined for segments of the smallest and highest probability density of the vibration signal. The reduction of the feature vector dimension was performed by means of the principal components analysis realised by singular value decomposition algorithm [16, 26]. In each maintenance state a series of 100 experiments was performed. Categorising of the tested state to the proper class as well as the classification quality analysis was done by means of the nearest neighbours method. The cross-validation technique was applied for the accuracy assessing. The classification accuracy was assessed on the ratio of the properly classified experimental results to their total number. All tested maintenance states were divided with 100% accuracy of the classification.

5. CONCLUSION

Investigations of the vehicle gearbox indicated non-Gaussian, heavy-tail character and long-term correlations of mechanical vibrations signals in the tested powertrain during the developing damage. Applying the dependence between parameters of the stable distributions and singularity spectra signals, the multifractal fluctuation, generalised entropy as well as the partition function of the recorded time series analyses, were also performed. The feature vector functioning as the data-driven empirical diagnostic model, was selected and verified. The performed investigations confirmed the possibility of assessing the observed states of the dynamic system, on the basis of the probability density distributions of mechanical vibrations signals, by means of parameters of their alpha-stable distribution and parameterised entropy. The proposed procedure of selection and classification can be realised within the vehicle on-board diagnostic system in the determined operating conditions. The currently continued research is focused on building experimental, non-linear diagnostic models and classification algorithms of the most often occurring mechanical defects in the vehicle powertrains.

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