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THE METHOD OF FREQUENCY DETERMINATION OF IMPULSE RESPONSE COMPONENTS BASED ON CROSS-CORRELATION VS. FAST FOURIER TRANSFORM

Adam KOTOWSKI

Faculty of Mechanical Engineering, Department of Automatic Control and Robotics
Bialystok University of Technology, ul. Wiejska 45C, 15-351 Bialystok, Poland

Summary

The paper presents results of impulse response spectral analysis that has been obtained using a method based on cross-correlation. The impulse response spectrum is achieved by correlating the impulse response and reference single-harmonic signals and using Hilbert transform to obtain an envelope of cross-correlation. Then, surface area under the envelope is calculating and its plot in frequency domain is making. The spectrum obtained this way has its advantage over the fast Fourier transform (FFT) that its spectral resolution does not depend on length of impulse response. At the same time, the spectral resolution can be much greater than spectral resolution resultant from FFT. Obtained results show that presented method gives possibilities to determine frequencies of impulse response components more accurate in comparison to FFT particularly for short-time impulse responses.

Keywords: impulse response, spectrum, peak frequency determination

METODA USTALANIA CZĘSTOTLIWOŚCI SKŁADOWYCH ODPOWIEDZI IMPULSOWEJ BAZUJĄCA NA FUNKCJI KORELACJI WZAJEMNEJ WZGLĘDEM SZYBKIEJ TRANSFORMATY FOURIERA

Streszczenie

Praca przedstawia wyniki analizy widmowej odpowiedzi impulsowej przy użyciu metody opartej o funkcję korelacji wzajemnej. Widmo odpowiedzi impulsowej uzyskiwane jest poprzez korelowanie odpowiedzi impulsowej z harmonicznym sygnałem odniesienia i zastosowaniu transformaty Hilberta w celu uzyskania obwiedni funkcji korelacji wzajemnej. Wówczas wyznaczane jest pole powierzchni pod obwiednią i jej wykres w funkcji częstotliwości. Tak uzyskane widmo posiada tą zaletę ponad szybką transformatę Fouriera (FFT), że jego rozdzielczość widmowa nie zależy od długości odpowiedzi impulsowej. Równocześnie, rozdzielczość widmowa potrafi być znacznie wyższa niż rozdzielczość widmowa wynikająca ze stosowania FFT. Uzyskane wyniki pokazują, że przedstawiona metoda stwarza możliwości aby ustalać częstotliwości składowych odpowiedzi impulsowej dokładniej w porównaniu do FFT.

Słowa kluczowe: odpowiedź impulsowa, widmo, wyznaczanie częstotliwości piku

1. INTRODUCTION

Impulse response as a result of impact testing has been used in wide area of engineering owing to its convenience and simplicity on experimentation [1]. It is well known that using fast Fourier transform (FFT) for spectrum analysis will give an immediate frequency profile of recorded signals. When analyzing signals using FFT, frequency resolution is fixed as an inverse of the duration of the analyzed signal [2, 8] and, as a result, it fails to meet the requirements of measurement. One of the ways to increase frequency resolution and improve frequency determination is interpolation. It improves the resolution by a few orders, depending on the interpolation method [6, 7]. In order to avoid limitation in frequency resolution using FFT, it has been investigated the problem of frequency resolution and showed that it was possible to obtain frequency resolution of one-tenth of the spacing between the frequency points produced by the Fourier transform [3]. For increasing spectral frequency resolution and improving frequency estimation, the zero-padding technique also is widely used [4, 10]. In general, the interpolation algorithms can be a computing-cost-effective replacement of the zero-padding technique in applications [7].

There are a great number of engineering applications of correlation function. Looking for its new applications, the cross-correlation function has been utilized to correlate real-measured signal and a single-harmonic signal generated by software. Also, the Hilbert transform was used to obtain the envelope of the cross-correlation function [11, 12]. In particular cases, for stationary signals, experimental results have shown a linear shape of the envelope. It is observed when correlated signals have a common frequency value [9]. This effect is well noted and very sensitive to generated single harmonic signal frequency.

This paper describes a method of determination of frequency of the harmonic developed on the basis of the cross-correlation function and its envelope. The main advantage of this method over the FFT is its ability to obtain different frequency resolution from that obtained by using FFT, often required as increased resolution, e.g. ten times increased.

2. METHODOLOGY

The cross-correlation function $R_{xy}(\tau)$ between two processes, x(t) and y(t), is calculating by the expression as follows [2]

$$R_{xy}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)y(t+\tau)\,dt \tag{1}$$

where: T – signal record length, τ – time delay.

The Hilbert Transform (HT) enables calculation of the envelope A(t) of a real-valued function x(t) as follows [5, 11]

$$A(t)=\sqrt{x^2(t)+\tilde{x}^2(t)} \tag{2}$$

where $\tilde{x}(t)$ is HT of a real-valued function x(t). Then, an envelope of the cross-correlation function R_{env} is as follows

$$R_{env}(\tau)=\sqrt{R_{xy}{}^2(\tau)+\tilde{R}_{xy}{}^2(\tau)} \tag{3}$$

where: $\tilde{R}_{xy}$ is HT of the cross-correlation R_{xy}.

The envelope of the cross-correlation of two harmonic processes with one of them being damped, decays exponentially. In addition, the equality of the frequencies of the harmonics of two processes creates the highest position of the envelope plot in comparison to the non-equality of the frequencies.

Apart from analyzed signal *y*(t), the method used in the paper requires a series of harmonic signals g_i(t) generated as follows:

$$g_i=\sin(2\pi\cdot w\cdot i\cdot t) \tag{4}$$

where: *i* - an integer value (index), *w* – frequency resolution.

After correlating the input and the generated signals, the plot position of the envelope of the cross-correlation indicates an identification of the harmonic detected. This phenomenon is easy to observe and the determination of common frequency is simple. As a result, the envelope plot can be effectively used to identify the harmonics incorporated in recorded signals with no need for the Fourier transform. The frequency value of the harmonic in the input signal *y*(t) can be determined using the envelope plot. Studying the envelopes, we obtain both *w* and *i* values of signal g_i(t) used for calculations. Thus, formula ($w{\cdot}i$) indicates the frequency of the identified harmonic. Apart from cross-correlation envelope plots, indicator Se is used to express envelope position numerically as follows:

$$S_e(n)=\Delta t\sum_{n=1}^{N}R_{env}(n) \tag{5}$$

where R_{env} is the cross-correlation envelope, Δt is the sampling period and *N* stands for the number of samples. As a result, it is possible to prepare the S_e indicator plot, as a result of spectral analysis, for a frequency span in which the impulse response has been recorded. The frequency resolution of S_e plot is strictly connected to *w* value (spectral resolution).

3. 3-DOF SYSTEM IMPULSE RESPONSE ANALYSIS

To illustrate the comparison between cross-correlation-based method (CCBM) and fast Fourier transform (FFT) for spectral analysis of impulse response, a numerical example of a 3-dof system impulse response is considered.

The unit impulse response function of a multi-degree-of-freedom system can be expressed as [1]

$$h(t)=\sum_{r=1}^{n}A_r\exp(-\sigma_r t)\sin(\omega_{dr}t) \tag{6}$$

where: A_r - the *r*th modal constant, σ_r - the *r*th modal damping, ω_{dr} - the *r*th damped angular frequency of the system. Thus, three-degree-of-freedom (3-dof) system impulse response can be expressed as:

$$\begin{aligned}h(t)&=A_1\exp(-\sigma_1 t)\sin(2\pi f_1 t)+\\&+A_2\exp(-\sigma_2 t)\sin(2\pi f_2 t)+\\&+A_3\exp(-\sigma_3 t)\sin(2\pi f_3 t)\end{aligned} \tag{7}$$

For obtaining time history of 3-dof system impulse response, values of parameters of Eq. (7) have been assumed. They are: A_1=0.25, A_2=0.70, A_3=0.15, σ_1=560, σ_2=610, σ_3=640, f_1=1556.4Hz, f_2=5231.8Hz, f_3=7684.5Hz.

The considered impulse response have been calculated by sampling frequency equal to f_s=32768(2^{15})Hz and length equal to N=512 samples. Thus, sampling time is equal to 0.0156 second. The impulse response h(t) under consideration is shown in Fig. 1.

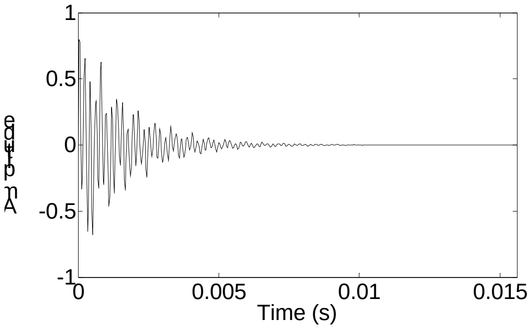

Fig. 1. 3-dof system impulse response

The spectral analysis using FFT has resulted frequencies readout by frequency resolution Δf as a result of the sampling frequency and number of samples N, $\Delta f_{FFT}=f_s/N$ [2, 8]. It has been Δf_{FFT}=32768/512=64Hz. The spectral analysis using CCBM has resulted frequencies readout by different frequency resolution independent of the sampling time of impulse response. The spectra obtained using CCBM and FFT are shown in Fig. 2.

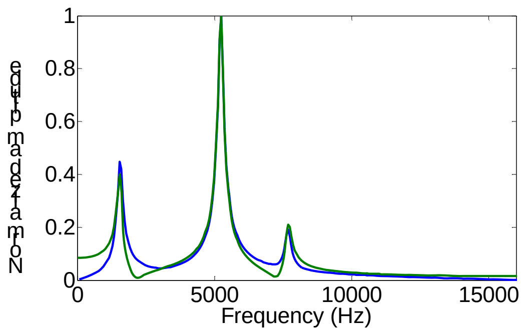

Fig. 2. Spectra obtained by CCBM (blue) and FFT (green)

The values of frequencies read from plots for both methods are shown in Tab. 1. There are three values for succeeding three plot peaks. The one of CCBM frequency resolution has been fixed the same as for FFT (64Hz). For further research of CCBM, resolution of 10Hz and 1Hz has also been considered for that method.

The best accuracy of frequency determination has been obtained for CCBM and frequency resolution equal to 1Hz, see Fig.3 – red bar. In this case, differences between real frequency and measured frequency for considered three peaks reached 2.6Hz, 0.2Hz and 0.5Hz. Using FFT, this differences are 20.4Hz, 16.2Hz and 4.5Hz respectively. Thus, the accuracy of frequency determination using CCBM are several times lower than using FFT.

Table 1. Frequency determination

Real frequency (Hz)	FFT Δf=64Hz (Hz)	CCBM Δf=64Hz (Hz)	CCBM Δf=10Hz (Hz)	CCBM Δf=1Hz (Hz)
1556.4	1536	1536	1560	1559
5231.8	5248	5248	5230	5232
7684.5	7680	7680	7680	7685

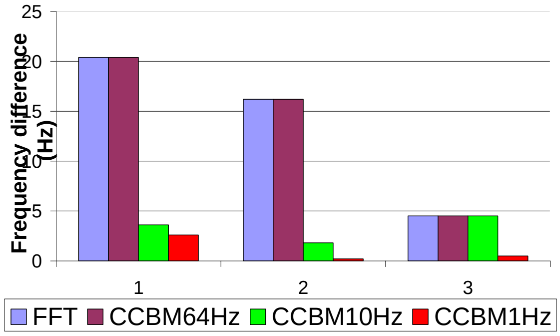

Fig. 3. Frequency differences between real and determined frequencies

4. SDOF SYSTEM DIAGNOSTIC CASE

In this section, stiffness change of single degree-of-freedom (SDOF) system has been considered. The SDOF system is presented in Fig. 4. It has been applied proposed cross-correlation-based method (CCBM) and fast Fourier transform (FFT) for obtaining the spectrum and damped frequency determination.

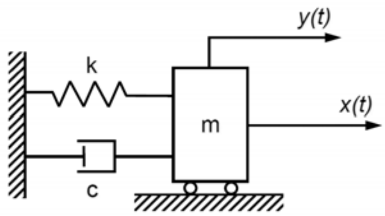

Fig. 4. SDOF system: m-mass, c- damping coefficient, k- stiffness.

The form of frequency response of SDOF system under impulse load is as follows [2]

$$H(f)=\frac{1/k}{1-(f/f_n)+j2\xi(f/f_n)} \tag{8}$$

in which undamped natural frequency is $f_n = 1/2\pi\sqrt{k/m}$ and damping ratio $\xi = c/2\sqrt{km}$.

For ξ<1, the impulse response function of the system described above is given by the inverse Fourier transform of Eq. 3 as follows [2]

$$h(t) = Ae^{-\delta t}\sin(2\pi f_d t) \tag{9}$$

where $A = 2\pi f_n^2 / kf_d$, $\delta = 2\pi f_n \xi$ and $f_d = f_n \sqrt{1-\xi^2}$.

In the example taken into consideration mass m and damping coefficient c and stiffness k are equal 1 kg and 250 Ns/m and $1.6\cdot10^6$ N/m, respectively. The value of damped natural frequency was calculated and equal to 200,33Hz. Also, impulse response was generated by sampling frequency equal to 4096Hz and number of samples N=256. The impulse responses of considered system is shown in Fig. 5.

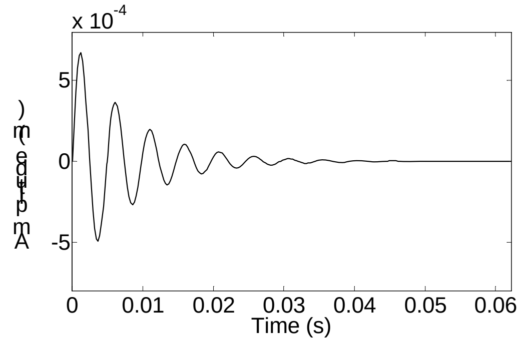

Fig. 5. SDOF system impulse response

In the case of using FFT, the spectral resolution Δf is 16Hz here. The use of CCBM allows spectral resolution Δf to be fixed arbitrary, e.g. 1Hz and 0.1Hz. For that CCBM approach and Δf=1Hz 16Hz this frequency is 202Hz for spectral resolution of 1Hz and 201.5Hz for spectral resolution of 0.1Hz.

To simulate stiffness change of SDOF system, stiffness k^* has been changed to 0.95k, 0.9k and 0.85k. This way, new values of real damped frequencies and determined frequencies have been obtained, see Tab. 2.

Table 2. Real and determined frequencies for k^* case

Real damped frequency (Hz)	**FFT Δf=16Hz (Hz)**	**CCEM Δf=1Hz (Hz)**	**CCEM Δf=0.1Hz (Hz)**
k*=k			
200.33	192	202	201.5
k*=0.95k			
195.21	192	197	196.5
Frequency shift			
5.12	0	5	5.0
k*=0.9k			
189.95	192	191	191.3
Frequency shift			
10.38	0	11	10.2
k*=0.85k			
184.54	176	186	186.0
Frequency shift			
15.79	16	16	15.5

Analyzing showed results, it is observed that using CCBM we can obtain much more accurate frequency determination than using FFT. For two cases (0.95k, 0.9k) frequency shift using FFT is zero when stiffness change of SDOF system can be detected in the form of decreasing of damped frequency by using CCBM. By k^*=0.85k, change of stiffness is detected by using both methods (FFT, CCBM).

Because of spectral resolution, frequency determined on the base of FFT not always has pointed the stiffness change. Using CCBM, spectral resolution can be improved (setting higher resolution) and change of SDOF system condition can by detected for lowest changes of stiffness.

4. DIAGNOSING THE ROTOR BLADE DAMAGE

Next diagnostic case is the case of notched rotor blade of 1st stage of an axial-flow compressor of aircraft jet engine (Fig. 6). The notch has been prepared near the base of the blade leading edge.

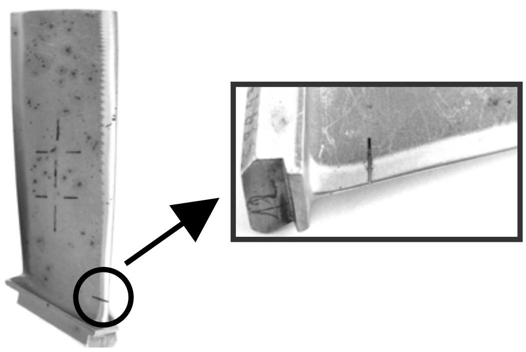
Fig. 6. Notched rotor blade

Acoustic impulse responses of both the intact and the damaged blade have been recorded by impulse load at middle zone of blade body. In every case it has been five hammer impacts resulting five impulse responses. The parameters of recording was: sampling frequency f_s=65536Hz, length of signal N=2048 samples. An exemplary acoustic impulse response of the tested rotor blade is shown in Fig. 7.

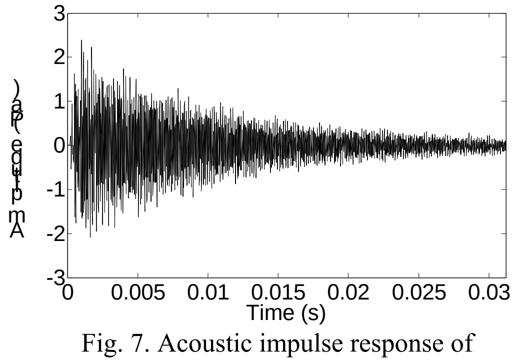

Fig. 7. Acoustic impulse response of the blade

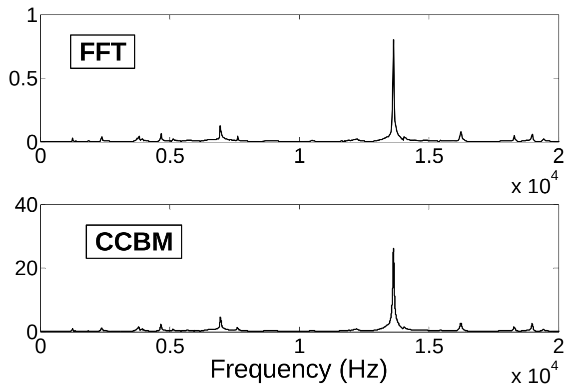

Fig. 8. Spectra obtained using FFT and CCBM

For diagnosing the blade damage, the change of frequency of harmonic at highest amplitude has been taken into consideration – Fig. 8.

When impulse responses of both the intact and the damaged blade were analyzed using FFT (32Hz spectral resolution), it has been obtained the same values of frequency of considered harmonic. It has always been 13632Hz. Whereas the use of CCBM enables the spectrum to have greater resolution than FFT (e.g. 1Hz) and frequency shift is observed as a result of blade damage. It is shown in Tab. 3.

Table 3. Frequency determination using CCBM by fixed 1Hz spectral resolution

No.	Intact blade	Damage blade
1	13641	13630
2	13639	13632
3	13639	13630
4	13640	13629
5	13639	13632
Mean	**13639,6**	**13630,6**
Std	**0,894**	**1,34**

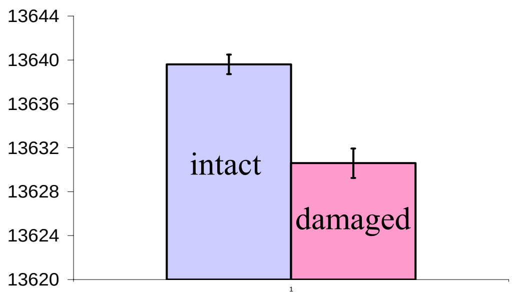

Fig. 9. Determined frequencies for intact blade and damaged blade using CCBM

The frequency shift using FFT was zero in spite of blade damage. Thus, frequency determination of impulse response components by using FFT have not been sufficient to detect defect of the blade. As shown in Fig. 9, the use of CCBM enables to observe the shift in frequency as a result of blade damage.

7. CONCLUSIONS

The paper presents a comparison of results of frequency determination of impulse response components that have been obtained using cross-correlation-based method to results obtained using fast Fourier transform. The proposed non-Fourier method of frequency determination of spectrum components is achieved by correlating the analyzed signal and reference single-harmonic signals and using Hilbert transform to obtain an envelope of cross-correlation function. The spectral analysis obtained by using proposed method has its advantage over the FFT that the spectral resolution does not depend on signal length and it gives a possibility to obtain a controllable spectral resolution. Moreover, the spectral resolution can be much greater than spectral resolution resultant from FFT. By extension, frequency readings are more accurate by doing proposed frequency resolution improvement. Thus, cross-correlation-based method can be an additional method for improving the accuracy of natural frequencies measurement using impulse tests.

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PhD. Eng. **Adam KOTOWSKI** is Assistant Professor in Faculty of Mechanical Engineering, Bialystok University of Technology. He received the MS degree in Automatic Control and Robotics, in 1999, and completed his PhD degree in Technical Diagnostics from Faculty of Mechanical Engineering, Bialystok University of Technology, in 2006. He is particularly interested in analysis of vibroacoustic signals.